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[illegible] :

1) [illegible] $(\mathrm{E},+,\cdot)$ [illegible] $(\mathrm{M}_2(\mathbb{R}),+,\cdot)$.

[illegible] O [illegible] E $\left(\mathrm{M}(0,0)=\mathrm{O}\right)$: $\mathrm{E}\neq\varnothing$.

[illegible] $(\lambda,\mu)\in\mathbb{R}^2$ [illegible] $\mathrm{M}(a,b)$ [illegible] $\mathrm{M}(c,d)$ [illegible] E :

$$\lambda\mathrm{M}(a,b)+\mu\mathrm{M}(c,d)=\mathrm{M}(\lambda a+\mu c,\lambda b+\mu d)$$

[illegible] : $\lambda\mathrm{M}(a,b)+\mu\mathrm{M}(c,d)\in\mathrm{E}$.

[illegible] $(\mathrm{E},+,\cdot)$ [illegible] $(\mathrm{M}_2(\mathbb{R}),+,\cdot)$.

[illegible] $\mathrm{M}(a,b)$ [illegible] E :

$$\mathrm{M}(a,b)=\begin{pmatrix} a & 0\\ 0 & a\end{pmatrix}+\begin{pmatrix} 0 & \sqrt{3}b\\ -\frac{1}{\sqrt{3}}b & 0\end{pmatrix}=a\mathrm{I}+b\mathrm{J}$$

[illegible] (I,J) [illegible] $(\mathrm{E},+,\cdot)$.

[illegible] (I,J) [illegible] .

[illegible] $(\alpha,\beta)\in\mathbb{R}^2$ [illegible] : $\alpha\mathrm{I}+\beta\mathrm{J}=\mathrm{O}$.

[illegible] : $\alpha\mathrm{I}+\beta\mathrm{J}=\mathrm{M}(\alpha,\beta)$ [illegible] : $\begin{pmatrix}\alpha & \sqrt{3}\beta\\ -\frac{1}{\sqrt{3}}\beta & \alpha\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

[illegible] : $\alpha=\sqrt{3}\beta=-\dfrac{1}{\sqrt{3}}\beta=0$ [illegible] : $\alpha=\beta=0$.

[illegible] (I,J) [illegible] E .

[illegible] (I,J) [illegible] $(\mathrm{E},+,\cdot)$.

2) [illegible] E [illegible] $(\mathrm{M}_2(\mathbb{R}),\times)$.

[illegible] $\mathrm{M}(a,b)$ [illegible] $\mathrm{M}(c,d)$ [illegible] E :

$$\mathrm{M}(a,b)\times\mathrm{M}(c,d)=(a\mathrm{I}+b\mathrm{J})\times(c\mathrm{I}+d\mathrm{J})=ac\mathrm{I}+(bc+ad)\mathrm{J}+bd\mathrm{J}^2$$

[illegible] : $\mathrm{J}^2=-\mathrm{I}$ [illegible] :

$$\mathrm{M}(a,b)\times\mathrm{M}(c,d)=(ac-bd)\mathrm{I}+(bc+ad)\mathrm{J}=\mathrm{M}(ac-bd,bc+ad)$$

[illegible] E [illegible] $(\mathrm{M}_2(\mathbb{R}),\times)$.

[illegible] f [illegible] $(\mathbb{C}^*,\times)$ [illegible] $(\mathrm{E}^*,\times)$.

[illegible] $z=a+ib$ [illegible] $z'=c+id$ [illegible] $\mathbb{C}^*$ [illegible] : $z\times z'=(ac-bd)+i(bc+ad)$

[illegible] : $f(z\times z')=\mathrm{M}(ac-bd,bc+ad)$

[illegible] : $f(z\times z')=\mathrm{M}(a,b)\times\mathrm{M}(c,d)=f(z)\times f(z')$.

[illegible] f [illegible] $(\mathbb{C}^*,\times)$ [illegible] $(\mathrm{E}^*,\times)$.

[illegible] f [illegible] .

[illegible] (I,J) [illegible] E [illegible] :

$$(\forall\mathrm{M}\in\mathrm{E}^*)\ ;\ \left(\exists!(a,b)\in\mathbb{R}^2-\{(0,0)\}\right)\ /\ \mathrm{M}=a\mathrm{I}+b\mathrm{J}$$

[illegible] : $a\mathrm{I}+b\mathrm{J}=\mathrm{M}(a,b)=f(a+ib)$ [illegible] :

$$(\forall\mathrm{M}\in\mathrm{E}^*)\ ;\ \left(\exists!z=a+ib\in\mathbb{C}^*\right)\ /\ \mathrm{M}=f(a+ib)=f(z)$$.

[illegible] f [illegible] $\mathbb{C}^*$ [illegible] E^* [illegible] .

3) [illegible] $(\mathrm{E},+,\times)$ [illegible] .

[illegible] $(\mathrm{E},+,\cdot)$ [illegible] $(\mathrm{E},+)$ [illegible] .

[illegible] $(\mathbb{C}^*,\times)$ [illegible] f [illegible] $(\mathbb{C}^*,\times)$ [illegible] $(\mathrm{E}^*,\times)$

[illegible] $(\mathrm{E}^*,\times)$ [illegible] .

[illegible] [illegible] [illegible]

[illegible] - 2008 [illegible]

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[illegible] $M_2(\mathbb{R})$ [illegible] E [illegible] $M_2(\mathbb{R})$ [illegible]

. E [illegible]

. $(E,+,\times)$ [illegible]

. $J\times X^3=I$: [illegible] E [illegible] (4

: [illegible]

$$(\forall X\in E^*);\ J\times X^3=I\Leftrightarrow f^{-1}(J\times X^3)=f^{-1}(I)\Leftrightarrow f^{-1}(J)\times f^{-1}(X^3)=f^{-1}(I)$$

$f^{-1}(X^3)=\left(f^{-1}(X)\right)^3$ [illegible] $f^{-1}(J)=i$ [illegible] $f^{-1}(I)=1$: [illegible]

: [illegible] $f^{-1}(X^3)=z^3$: [illegible] $z=x+iy$ [illegible] $X=M(x,y)$ [illegible]

$(z-i)\times(z^2+iz-1)=0$ [illegible] $z^3=-i=i^3$ [illegible] $i\times z^3=1$

. $z_2=\frac{\sqrt{3}-i}{2}$ [illegible] $z_1=-\frac{\sqrt{3}+i}{2}$ [illegible] $z_0=i$: [illegible]

: [illegible] E [illegible] $J\times X^3=I$ [illegible]

. $S=\left\{ M(0,1)=J\ ;\ M\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)\ ;\ M\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right) \right\}$

[illegible] : ■

. $(G): iz^2+(a+\bar{a}-i)z-\bar{a}-ia\bar{a}=0$: [illegible] $\mathbb{C}$ [illegible] -I

: (G) [illegible] **(1 -**

$$\Delta=(a+\bar{a}-i)^2+4i(\bar{a}+ia\bar{a})=(a+\bar{a})^2-2i(a+\bar{a})-1+4i\bar{a}-4a\bar{a}$$

$$=a^2+2a\bar{a}+\bar{a}^2-2ia-2i\bar{a}+4i\bar{a}-4a\bar{a}-1$$

$$=a^2-2a\bar{a}+\bar{a}^2-2i(a-\bar{a})-1$$

$$\Delta=(a-\bar{a})^2-2i(a-\bar{a})+i^2=(a-\bar{a}-i)^2$$: [illegible]

: [illegible] $\mathbb{C}$ [illegible] $\Delta=(a-\bar{a}-i)^2$: [illegible] (G) [illegible]

$$z_1=\frac{i-a-\bar{a}+a-\bar{a}-i}{2i}=-\frac{2\bar{a}}{2i}=i\bar{a}$$

. $z_2=\frac{i-a-\bar{a}-a+\bar{a}+i}{2i}=\frac{2(i-a)}{2i}=1+ia$ [illegible]

. $1+ia=a$ [illegible] $a=i\bar{a}$ [illegible] (G) [illegible] a [illegible] **(2**

$1+ia=a\Leftrightarrow a=\frac{1}{1-i}=\frac{1+i}{2}$: [illegible]

$a=i\bar{a}\Leftrightarrow \mathrm{Re}(a)+i\,\mathrm{Im}(a)=\mathrm{Im}(a)+i\,\mathrm{Re}(a)\Leftrightarrow \mathrm{Re}(a)=\mathrm{Im}(a)$ [illegible]

. $\mathrm{Re}(a)=\mathrm{Im}(a)$ [illegible] (G) [illegible] a : [illegible]

. $\mathrm{Re}(a)\neq\mathrm{Im}(a)$ [illegible] $a\in\mathbb{C}$ [illegible] $Z=\frac{(1+ia)-a}{(i\bar{a})-a}$: [illegible] **(1** -II

$\bar{Z}=\overline{\left(\frac{(1+ia)-a}{(i\bar{a})-a}\right)}=\frac{1-i\bar{a}-\bar{a}}{-ia-\bar{a}}=\frac{i+\bar{a}-i\bar{a}}{a-i\bar{a}}=\frac{(i-1)\bar{a}-i}{i\bar{a}-a}$: [illegible]

$Z=\bar{Z}$ [illegible] $Z\in\mathbb{R}$ [illegible] $C(1+ia)$ [illegible] $B(i\bar{a})$ [illegible] $A(a)$ [illegible]

$(1+ia)-a=(i-1)\bar{a}-i$: [illegible] $\frac{(1+ia)-a}{i\bar{a}-a}=\frac{(i-1)\bar{a}-i}{i\bar{a}-a}$: [illegible]

$\mathrm{Im}(a)=\frac{1}{2}$: [illegible] $2i\,\mathrm{Im}(a)=\frac{(1+i)^2}{2}$: [illegible] $a-\bar{a}=\frac{1+i}{1-i}$: [illegible]

. $(1+i)^2=2i$: [illegible]

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(2 $\text{Im}(a) \neq \frac{1}{2}$.

$R_1(B) = B' \Leftrightarrow z_{B'} - z_A = e^{-i\frac{\pi}{2}}(z_b - z_A) \Leftrightarrow b' - a = -i(i\overline{a} - a)$

$b' = a - i^2\overline{a} + ia = (1+i)a + \overline{a}$.

$R_2(C) = C' \Leftrightarrow z_{C'} - z_A = e^{i\frac{\pi}{2}}(z_C - z_A) \Leftrightarrow c' - a = i(1 + ia - a)$

$c' = a + i - a - ia = i(1-a)$.

$c' = i(1-a)$, $b' = (1+i)a + \overline{a}$.

$c' - b' = i - ia - a - ia - \overline{a} = i(1-2a) - (a + \overline{a})$

$z_E = \dfrac{z_B + z_C}{2} = \dfrac{1 + ia + i\overline{a}}{2} = \dfrac{1 + i(a+\overline{a})}{2}$: $[BC]$ E

$z_E - z_A = \dfrac{1 + i(a+\overline{a})}{2} - a = \dfrac{1 - 2a + i(a+\overline{a})}{2}$

$2i(z_E - z_A) = c' - b'$: $\dfrac{c' - b'}{z_E - z_A} = 2i$

$(AE) \perp (B'C')$: $\dfrac{c' - b'}{z_E - z_A} \in i\mathbb{R}$.

$\dfrac{B'C'}{AE} = \dfrac{|c' - b'|}{|z_E - z_A|} = \left|\dfrac{c' - b'}{z_E - z_A}\right| = |2i| = 2$: $B'C' = 2AE$.

I- $\mathbb{Z}^2$ (E): $35u - 96v = 1$.

(1 $35 \times 11 - 96 \times 4 = 385 - 384 = 1$: $(11,4)$ (E) .

(2 (u,v) (E) : $35(u-11) - 96(v-4) = 0$

$35(u-11) = 96(v-4)$ $(*)$: $96 \mid 35(u-11)$

$96 \wedge 35 = 1$: $96 \mid (u-11)$

$u = 11 + 96k \ / \ k \in \mathbb{Z}$

$(*)$: $v = 4 + 35k$

(E) : $S = \{ (11+96k, 4+35k) \ / \ k \in \mathbb{Z} \}$.

II- $\mathbb{N}$ (F): $x^{35} \equiv 2 \ [97]$.

(1 x (F) .

p $p^2 \leq 97$ 97 :

97	p	q	r
	2	48	1
	3	32	1
	5	19	2
	7	13	5

97

p :

$p^2 \leq 97$.

$d = x \wedge 97$.

$d = 97$, $d = 1$: 97 .

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$d = 97$ [illegible] $x \wedge 97 = 97$ [illegible] 97 [illegible] x

. $x^{35} \equiv 2\ [97]$ [illegible] $x^{35} \equiv 0\ [97]$ [illegible] x^{35} [illegible] 97 [illegible]

. [illegible] 97 [illegible] x [illegible] $d = 1$ [illegible]

$x^{97-1} \equiv 1\ [97]$ [illegible] $x \wedge 97 = 1$ [illegible] 97 [illegible]

. $x^{96} \equiv 1\ [97]$ [illegible]

$x^{35\times 11} \equiv 2^{11}\ [97]$ [illegible] $x^{35} \equiv 2\ [97] \Rightarrow \left(x^{35}\right)^{11} \equiv 2^{11}\ [97]$ [illegible]

$x^{(1+96\times 4)} \equiv 2^{11}\ [97]$ [illegible] $35\times 11 = 1 + 96\times 4$ [illegible]

$x \equiv x^{(1+99\times 4)}\ [97]$ [illegible] **(** [illegible] **)** $x^{96} \equiv 1\ [97]$ [illegible]

. $x \equiv 2^{11}\ [97]$ [illegible]

$x^{35} \equiv 2^{11\times 35}\ [97]$ [illegible] $x \equiv 2^{11}\ [97]$ [illegible] x [illegible] **- (2**

([illegible] **)** $2 \wedge 97 = 1 \Rightarrow 2^{96} \equiv 1\ [97]$ [illegible] $35\times 11 = 1 + 96\times 4$ [illegible]

. (F) [illegible] x [illegible] $x^{35} \equiv 2\ [97]$ [illegible] $2^{11\times 35} \equiv 2\ [97]$ [illegible]

. [illegible] (F) [illegible] $x \equiv 2^{11}\ [97]$ [illegible] **- (3**

. $2^{11} \equiv 11\ [97]$ [illegible] $2^{11} = 2048 = 21\times 97 + 11$ [illegible]

. $x \equiv 2^{11}\ [97] \Leftrightarrow x \equiv 11\ [97] \Leftrightarrow x = 11 + 97k / k \in \mathbb{N}$ [illegible]

. [illegible]

[illegible] :

. $f(x) = 2x - e^{-x^2}$ [illegible] $\mathbb{R}_+$ [illegible]

. $\lim\limits_{x\to+\infty}\left(f(x) - 2x\right)$ [illegible] **- (1**

. $\lim\limits_{x\to+\infty}\left(f(x)-2x\right) = \lim\limits_{x\to+\infty} -e^{-x^2} = -\lim\limits_{t\to-\infty} e^{-t} = 0$ [illegible]

. $y = 2x$ [illegible] $+\infty$ [illegible] (C) [illegible]

: $f'(x)$ [illegible]

$$(\forall x \in \mathbb{R}_+)\ ;\ f'(x) = (2x)' - \left(-x^2\right)' e^{-x^2} = 2 + 2xe^{-x^2} = 2\left(1 + xe^{-x^2}\right)$$

($(\forall x \in \mathbb{R}_+)\ ;\ xe^{-x^2} \geq 0$ [illegible] **)** $(\forall x \in \mathbb{R}_+)\ ;\ f'(x) > 0$ [illegible]

: [illegible] $\mathbb{R}_+$ [illegible] f [illegible]

x	0	$+\infty$
$f'(x)$	$+$	
f	-1 ↗	$+\infty$

$$\left(\lim_{x\to+\infty} 2x = +\infty \text{ و } \lim_{x\to+\infty} e^{-x^2} = 0\right) \Rightarrow \lim_{x\to+\infty} f(x) = +\infty$$

: [illegible] $\mathbb{R}_+$ [illegible] f [illegible] $\mathbb{R}_+$ [illegible]

$$J = f(\mathbb{R}_+) = \left[f(0); \lim_{x\to+\infty} f(x)\right[= [-1; +\infty[$$

. $f(\alpha) = 0$ [illegible] $\mathbb{R}_+$ [illegible] α [illegible] $0 \in J$ [illegible]

. $0 < \alpha < 1$ [illegible] $f(1) = 2 - \dfrac{1}{e} > 0$ [illegible] $f(0) = -1 < 0$ [illegible]

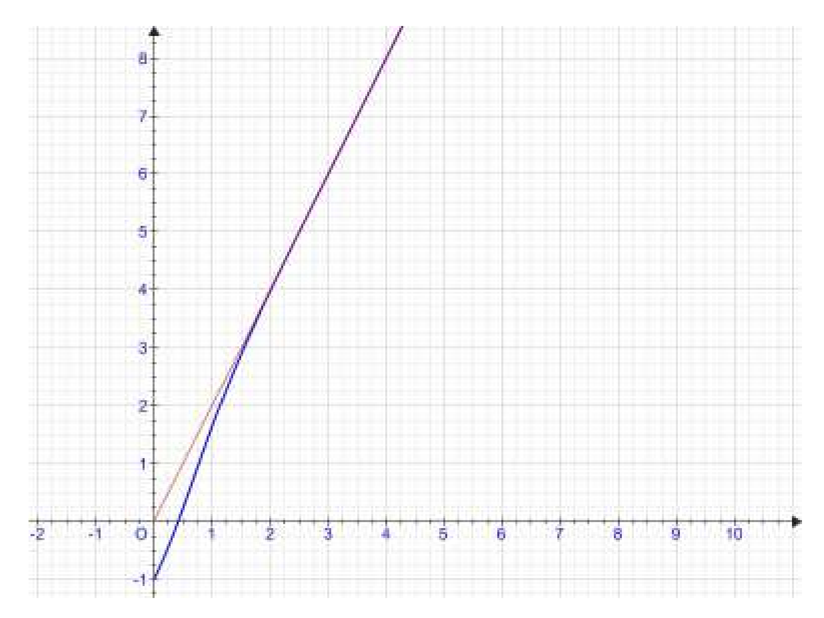
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$. \ [0;1]$ [illegible] $f(x)$ [illegible] **- ℓ**

$: \ 0 < \alpha < 1$ [illegible] α [illegible] $[0;1]$ [illegible] f

$. \ \left(\forall x \in]\alpha;1]\right) \ ; \ f(x) > 0 \ \leftrightarrow \left(\forall x \in [0;\alpha[\right) \ ; \ f(x) < 0$

$. \ (C)$ [illegible] **- (2**



$:$ [illegible] $\mathbb{R}_+$ [illegible] $g \leftrightarrow \varphi$ [illegible] **II**

$$. \ g(x) = x^2 - \int_0^x e^{-t^2}dt \ \leftrightarrow \begin{cases} \varphi(x) = \dfrac{1}{x}\displaystyle\int_0^x e^{-t^2}dt \ ; \ x > 0 \\ \varphi(0) = 0 \end{cases}$$

$:$ [illegible] $\mathbb{R}_+^*$ [illegible] $\mathbb{R}_+$ [illegible] $F : u \mapsto \int_0^u e^{-t^2}dt$ [illegible] **- (1**

$. \ \left(\forall x \in \mathbb{R}_+^*\right) \ ; \ F'(x) = e^{-x^2}$

$: \ \mathbb{R}_+^*$ [illegible] $[0;x]$ [illegible]

$. \ F(x) - F(0) = x.F'(c) \ :$ [illegible] $]0;x[$ [illegible] c [illegible]

$. \ \int_0^x e^{-t^2}dt - 0 = x.e^{-c^2} \ :$ [illegible]

$. \ \left(\forall x \in \mathbb{R}_+^*\right) \ ; \ \left(\exists c \in]0;x[\right) \ / \ \dfrac{1}{x}\int_0^x e^{-t^2}dt = e^{-c^2} \ :$ [illegible]

$\left(\exists c \in]0;1[\right) \ / \ \int_0^1 e^{-t^2}dt = e^{-c^2} \ :$ [illegible] $x = 1$ [illegible]

$. \ \int_0^1 e^{-t^2}dt < 1 \ : \ $ [illegible] $\ 0 < c < 1 \Rightarrow e^{-c^2} < e^{-0^2} = 1 \ :$ [illegible]

$. \ \left(\forall x \in \mathbb{R}_+\right) \ ; g(x) = \int_0^x f(t)dt \ :$ [illegible] **- (2**

$. \ \left(\forall x \in \mathbb{R}_+\right) \ ; \int_0^x f(t)dt = \int_0^x (2t)dt - \int_0^x e^{-t^2}dt = \left[t^2\right]_0^x - \int_0^x e^{-t^2}dt = g(x) \ :$ [illegible]

$\mathbb{R}_+$ [illegible] $x \mapsto \int_0^x f(t)dt \ :$ [illegible] $\mathbb{R}_+$ [illegible] f [illegible]

$\left(\forall x \in \mathbb{R}_+\right) \ ; \ \left(\int_0^x f(t)dt\right)' = f(x) \ :$ [illegible]

$. \ \left(\forall x \in \mathbb{R}_+\right) \ ; \ g'(x) = f(x) \ \leftrightarrow \mathbb{R}_+$ [illegible] g [illegible]

$]\alpha;1[$ [illegible] g [illegible] $\left(\forall x \in]\alpha;1[\right) \ ; \ g'(x) = f(x) > 0 \ :$ [illegible]

$\left(\forall t \in [0;\alpha[\right) \ ; \ f(t) < 0 \Rightarrow g(\alpha) = \int_0^\alpha f(t)dt < 0 \ :$ [illegible]

$. \ \int_0^1 e^{-t^2}dt < 1 \Rightarrow g(1) = 1^2 - \int_0^1 e^{-t^2}dt > 0 \ \leftrightarrow$

$. \ g(\beta) = 0 \ :$ [illegible] $]\alpha;1[$ [illegible] β [illegible]

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(3 -

$\mathbb{R}_+^*$ x

$e^{-x^2} \le e^{-t^2} \le 1$: $-x^2 \le -t^2 \le 0$: $[0;x]$

$$\int_0^x e^{-x^2}dt \le \int_0^x e^{-t^2}dt \le \int_0^x 1dt$$

$$xe^{-x^2} \le \int_0^x e^{-t^2}dt \le x \; .$$

$$(\forall x\in\mathbb{R}_+^*)\ ;\ e^{-x^2} \le \varphi(x) = \frac{1}{x}\int_0^x e^{-t^2}dt \le 1$$

$\lim\limits_{x\to0^+} e^{-x^2}=1$: $\lim\limits_{x\to0^+}\varphi(x)=1=\varphi(0)$

$$(\forall x\in\mathbb{R}_+^*)\ ;\ \varphi(x)=e^{-x^2}+\frac{2}{x}\int_0^x t^2e^{-t^2}dt$$

$\begin{cases} u(t)=e^{-t^2} \\ v'(t)=1 \end{cases}$: $\begin{cases} u'(t)=-2te^{-t^2} \\ v(t)=t \end{cases}$

$\int_0^x e^{-t^2}dt = \left[te^{-t^2}\right]_0^x + 2\int_0^x t^2e^{-t^2}dt$: $\frac{1}{x}$

$$\varphi(x)=e^{-x^2}+\frac{2}{x}\int_0^x t^2e^{-t^2}dt \; .$$

$\mathbb{R}_+$ $t\mapsto t^2e^{-t^2}$; $\mathbb{R}_+$ $x\mapsto\int_0^x t^2e^{-t^2}dt$

$$(\forall x\in\mathbb{R}_+)\ ;\ \left(\int_0^x t^2e^{-t^2}dt\right)' = x^2e^{-x^2} \; .$$

$x\mapsto \frac{2}{x}$, $x\mapsto e^{-x^2}$ $\mathbb{R}_+^*$; φ $\mathbb{R}_+^*$

$$(\forall x\in\mathbb{R}_+^*)\ ;\ \varphi'(x) = \left(e^{-x^2}\right)' + \left(\frac{2}{x}\right)'\int_0^x t^2e^{-t^2}dt + \frac{2}{x}\left(\int_0^x t^2e^{-t^2}dt\right)'$$

$$(\forall x\in\mathbb{R}_+^*)\ ;\ \varphi'(x) = -2xe^{-x^2} - \frac{2}{x^2}\int_0^x t^2e^{-t^2}dt + \frac{2}{x}x^2e^{-x^2}$$

$$(\forall x\in\mathbb{R}_+^*)\ ;\ \varphi'(x) = -\frac{2}{x^2}\int_0^x t^2e^{-t^2}dt \; .$$

φ $[0;1]$ $\mathbb{R}_+$

$$(\forall x\in\mathbb{R}_+^*)\ ;\ \varphi'(x) < 0$$

$$\varphi([0;1]) = [\varphi(1);\varphi(0)] = [\varphi(1);1] \; .$$

$[\varphi(1);1]\subset[0;1]$: $0<\varphi(1)=\int_0^1 e^{-t^2}dt<1$

$$\varphi([0;1])\subset[0;1] \; .$$

(4 - $(\forall x\in\mathbb{R}_+)\ ;\ \int_0^x t^2e^{-t^2}dt \le \frac{x^3}{3}$.

$(\forall t\in[0;x])\ ;\ t^2e^{-t^2}\le t^2$: $e^{-t^2}\le 1$: $[0;x]$

$$(\forall x\in\mathbb{R}_+)\ ;\ \int_0^x t^2e^{-t^2}dt \le \int_0^x t^2dt = \left[\frac{t^3}{3}\right]_0^x = \frac{x^3}{3} \; .$$

$$(\forall x\in\mathbb{R}_+^*)\ ;\ \left|\varphi'(x)\right| = \frac{2}{x^2}\int_0^x t^2e^{-t^2}dt$$

$(\forall x\in\mathbb{R}_+)\ ;\ \int_0^x t^2e^{-t^2}dt \le \frac{x^3}{3}$: $\frac{2}{x^2}$

$$(\forall x\in\mathbb{R}_+^*)\ ;\ \left|\varphi'(x)\right| \le \frac{2}{3}x \; .$$

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$\left(\forall x\in\left]0;1\right[\right);\ \left|\varphi'(x)\right|\le\frac{2}{3}$: [illegible] . ($x\in\left]0;1\right[\Rightarrow\frac{2}{3}x\le\frac{2}{3}$: [illegible])

[illegible] $\mathbb{R}_+^*$ [illegible] :

$$\varphi(x)=x\Leftrightarrow\frac{1}{x}\int_0^x e^{-t^2}dt=x\Leftrightarrow\int_0^x e^{-t^2}dt=x^2\Leftrightarrow x^2-\int_0^x e^{-t^2}dt=0\Leftrightarrow g(x)=0$$

$\left(\forall x\in\mathbb{R}_+^*\right);\varphi(x)=x\Leftrightarrow g(x)=0$: [illegible] .

5) [illegible] $(u_n)_{n\ge0}$ [illegible] :

$u_0=\frac{2}{3}$ [illegible] $(\forall n\in\mathbb{N});\ u_{n+1}=\varphi(u_n)$.

4 [illegible] $(\forall n\in\mathbb{N});\ 0\le u_n\le1$: [illegible] .

[illegible] $n=0$ [illegible] : $0\le u_0=\frac{2}{3}\le1$.

[illegible] : $0\le u_n\le1$

[illegible] : $\varphi([0;1])\subset[0;1]$ [illegible] : $u_{n+1}=\varphi(u_n)\in[0;1]$

[illegible] : $0\le u_{n+1}\le1$.

[illegible] : $(\forall n\in\mathbb{N});\ 0\le u_n\le1$.

[illegible] $n=0$ [illegible] :

$u_0=\frac{2}{3}\in[0;1]$ [illegible] $\beta\in[0;1]$ [illegible] : $|u_0-\beta|\le|1-0|=1=\left(\frac{2}{3}\right)^0$.

[illegible] : $|u_n-\beta|\le\left(\frac{2}{3}\right)^n$.

[illegible] : $g(\beta)=0$ [illegible] 4) - [illegible] : $\varphi(\beta)=\beta$.

[illegible] : $|u_{n+1}-\beta|=|\varphi(u_n)-\varphi(\beta)|$

[illegible] u_n [illegible] β [illegible] :

$$|\varphi(u_n)-\varphi(\beta)|\le\frac{2}{3}|u_n-\beta|$$

[illegible] $|u_n-\beta|\le\left(\frac{2}{3}\right)^n$ [illegible] $|u_{n+1}-\beta|\le\left(\frac{2}{3}\right)^{n+1}$.

[illegible] : $(\forall n\in\mathbb{N})\ ;\ |u_n-\beta|\le\left(\frac{2}{3}\right)^n$.

[illegible] $(u_n)_{n\ge0}$ [illegible] .

[illegible] $0<\frac{2}{3}<1$ [illegible] : $\lim_{n\to+\infty}\left(\frac{2}{3}\right)^n=0$

[illegible] : $(\forall n\in\mathbb{N})\ ;\ |u_n-\beta|\le\left(\frac{2}{3}\right)^n$ [illegible] $(u_n)_{n\ge0}$ [illegible]

[illegible] β .

[illegible] :

[illegible] .

[illegible] .